Notes on externality regulation and imperfect competition

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June 20, 2014

Abstract

In these notes, we review some of the classical contributions to the theory of externality regulation under imperfect competition. The main papers covered are Buchanan (1962) and Lee (1975). A comprehensive and fairly recent review of the literature can be found in Requate (2007).

1 Pigovian taxes an externalities : debunking common wisdom

A very much ingrained common wisdom regarding externalities is that they can be dealt with through so-called Pigovian taxation. If anything, this is probably what a typical undergraduate student remembers from the classical ECON101 textbook treatment of externalities. A good student might even remember the "golden rule" of Pigovian taxation : let the marginal tax rate equal the difference between the marginal private cost and the marginal social cost (for the case of a negative externality). As often, the result that Pigovian taxes are effective at regulating externalities has been unduly generalized, way beyond the restricted set of assumptions on which its validity relies. It now serves as a cheap argument to justify higher tax rates on virtually any good which can be associated with a negative external effect.[1]

Buchanan fiercely opposed such blind application of the Pigovian doctrine. He knew that Pigou taxes could be effective at dealing with externalities in competitive settings. But he clearly saw that under different market structures, this needed not be the case.[2] His graphical argument developed in Buchanan (1969) is based on a particularly simple example which we reproduce in figure 1.

Consider the standard partial equilibrium framework from undergraduate textbooks. Suppose that we are equipped with the traditional *Marshallian surplus* notion of social welfare (see frame I). For depiction simplicity, assume that PMc the private marginal cost of the monopolist is constant (see figure 1). Assume that production induces a negative externality perfectly proportional to the quantity produces. As a consequence, the social marginal cost SMc is higher than the the private marginal cost, and the monetary value of the externality per quantity produced is precisely $SMc - PMc$.

Frame I : Marshallian surplus, a primer

The literature which we review in these notes relies heavily on the partial equilibrium notion of *Marshallian surplus* to compare different allocations. The Marshallian surplus is a particular way of measuring social welfare. Intuitively, it measures the monetary benefit agents (consumers and producers) draw from the existence of the market at equilibrium. Because we are discussing externalities, we will have to deduct from these benefits the external cost induced by the production/consumption of the equilibrium quantity. From a practical point of view, the Marshallian surplus is simply the "areas under/above the demand/supply curves" from undergraduate economics. But it may worth reviewing its mechanics and theoretical underpinning. Chiefly, it matters to keep in mind the assumptions on individual preferences Marshallian surpluses relies on.

[1] See for instance Mankiw's argument on his blog post "The Pigou Club Manifesto". I do not claim that Mankiw's case for higher gasoline taxes cannot be backed by a rigorous taxation analysis. I am just noticing the use of the generic argument "externalities justify taxes" without further reference to the specificities of the gasoline market.

[2] See Buchanan (1962) for another line of qualifications on the desirability of Pigovian taxes.

Marshallian surplus, a primer (continued)

The partial equilibrium framework. In a partial equilibrium framework, we focus on the market for a single good q. It is assumed that this market is small with respect to the rest of the economy. In particular, what happens on this market does not influence the equilibrium prices on any other market. This allows to study the market for q in isolation, without having to care about interactions with other markets (as we would in a general equilibrium framework).

The firms. For simplicity, consider the case of a single or representative firm (more on the multiple firm case in Frame II). The firm produces q out of a set of inputs $(x_1, \ldots, x_n)$. The quantity q is related to the quantity of inputs used through the production function

$$q = f(x_1, \ldots, x_n). \tag{1}$$

Given our partial equilibrium assumptions, the prices of inputs are fixed, say $\boldsymbol{w} \coloneqq (w_1, \ldots, w_n)$. No matter how much of input x_i the firm uses to produce q, these quantities are small enough not to affect the market of x_i. Suppose that at an equilibrium the total quantity produced is $\tilde{q}$. One could wonder what is the minimum cost for the firm to produce $\tilde{q}$. Given the prevalent input prices, we would like to find the cheapest input combinations $\boldsymbol{x} \coloneqq (x_1, \ldots, x_n)$ that allow the firm to produce $\tilde{q}$. For any $\tilde{q}$ this minimum cost is defined by the cost function

$$c(\tilde{q}, \boldsymbol{w}) = \min_{\boldsymbol{x}} \quad \sum_{i=1}^{n} w_i x_i \qquad s.t. \qquad f(\boldsymbol{x}) = \tilde{q} \tag{2}$$

If in addition the equilibrium price is $\tilde{p}$, we have that the equilibrium firm profit is

$$\pi(\tilde{p}, \tilde{q}; \boldsymbol{w}) = \max_{\boldsymbol{x}} \tilde{p}\tilde{q} - \sum_{i=1}^{n} w_i x_i = \tilde{p}\tilde{q} - c(\tilde{q}, \boldsymbol{w}). \tag{3}$$

In Marshallian terms, the firm's profit is sometimes called *producer surplus* and denoted $PS(\tilde{p}, \tilde{q}; \boldsymbol{w}) \coloneqq \pi(\tilde{p}, \tilde{q}, ; \boldsymbol{w})$.

The consumers. We want to find a way to compare the producer's surplus with the benefit that the consumers derive from the equilibrium. As the producer's surplus is expressed in monetary terms, we need to find a way to express consumer's benefits in monetary terms too.This can be done under different sets of assumptions. Here we present one based on a continuum of consumers. (see (Varian, 1992, chapt. 10) for an alternative approach).
Each consumer comes on the market for good q to buy one unit of the good. The consumer will buy if the price is below some reservation price, and refrain from buying otherwise. Formally, consumer h's utility is assumed to take the linear form

$$u_h = \begin{cases} m + \theta_h - p_q, & \text{if } q_h = 1 \\ m, & \text{if } q_h = 0 \end{cases}, \tag{4}$$

where m is the h's wealth she uses to buy q as well as other goods. One can see that the consumer will buy a unit of the good if and only if $\theta_h \leq p_q$, her reservation price. We will assume that there have a mass M of consumers who differ only by their willingness to pay θ_h. Their willingness to pay is distributed according to some distribution $\theta_h \sim F(\theta_h)$ with support $[0, \bar{\theta}]$ for some $\bar{\theta} > 0$. Notice that given the reservation price behavior, the aggregate inverse demand is conveniently represented by

$$P(q) = F^{-1}\left(\frac{q}{M}\right). \tag{5}$$

In other words, given some quantity q, the price that clears market is $F^{-1}\left(\frac{q}{M}\right)$. In effect, the aggregate demand at $F^{-1}\left(\frac{q}{M}\right)$ is $MF(F^{-1}\left(\frac{q}{M}\right)) = q$. Notice also that for every q, $P(q)$ is the

Marshallian surplus, a primer (continued)

willingness to pay of the agents who are exactly indifferent between buying and not buying at $P(q)$. These consumers are called the *marginal consumers at q*.

Now suppose the equilibrium quantity is $\tilde{q}$ and the corresponding equilibrium price $\tilde{p}$. For every $q \leq \tilde{q}$, $P(q) - \tilde{p}$ is the monetary benefit that the marginal consumer at q draws from the market. Given the consumers' utility function (4), the marginal consumer at q would in fact be indifferent between paying a fee $P(q) - \tilde{p}$ to access to the market, and not not having access to the market at all. This sounds like a meaningful way to measure the benefit consumer's derive from the market at equilibrium. For some $\tilde{q}$, if we integrate $P(q) - \tilde{p}$ up to $\tilde{q}$, we sum this benefit for all the agents buying a part of the aggregate production $\tilde{q}$. This is the notion of *consumer's surplus*:

$$CS(\tilde{q}, \tilde{p}) \coloneqq \int_0^{\tilde{q}} P(q) - \tilde{p} \;\; dq \tag{6}$$

Marshallian surplus. Because we have expressed consumer surplus in monetary terms, it is now commensurable with producer surplus (remember producer surplus = profit at equilibrium). When there are externalities, we still need to account for the external costs and benefits induced by production. Again, this needs to be done in monetary terms if we are to be able to compare it with consumer and producer surplus. For simplicity, consider only the case of negative externalities. We will assume that there exists a function $D(q)$ representing the monetary equivalent for consumers of the external damages induced by the production of q. The construction of this function is not unproblematic, but we will not address it here. In first approximation it can be thought of as the amount of money that agents (consumer and firms) would be ready to pay to get rid of the externality.

Given input prices $\boldsymbol{w}$, the Marshallian surplus at some equilibrium allocation $(\tilde{p}, \tilde{q})$ is then the consumer surplus, plus the producer surplus, minus the external effect $D(q)$,

$$MS(\tilde{p}, \tilde{q}; \boldsymbol{w}) \coloneqq CS(\tilde{q}, \tilde{p}) + PS(\tilde{q}, \tilde{p}; \boldsymbol{w}) - D(\tilde{q}). \tag{7}$$

One might object that we should only care about the consumer surplus and not about producer surplus. But profits are eventually redistributed to consumers in one way or another. Given our assumption on the consumer's utility function and the partial equilibrium framework we work in, producer surplus actually is a form of consumer surplus. Finally, notice that Marshallian surplus can be reformulated as

$$\begin{aligned} MS(\tilde{p}, \tilde{q}; \boldsymbol{w}) &\coloneqq CS(\tilde{q}, \tilde{p}) + PS(\tilde{q}, \tilde{p}; \boldsymbol{w}) - D(\tilde{q}). \\ &= \int_0^{\tilde{q}} (P(q) - \tilde{p}) \; dq + (\tilde{p}\tilde{q} - c(\tilde{q}, \boldsymbol{w})]) - D(\tilde{q}) \\ &= \int_0^{\tilde{q}} P(q) \; dq - \tilde{p}\tilde{q} + \tilde{p}\tilde{q} - c(\tilde{q}, \boldsymbol{w}) - D(\tilde{q}) \\ &= \int_0^{\tilde{q}} P(q) \; dq - c(\tilde{q}, \boldsymbol{w}) - D(\tilde{q}). \end{aligned} \tag{8}$$

That is Marshallian surplus is the sum of consumers' willingnesses to pay minus the cost of production and the cost of externalities.

Consider first a competitive setting. At equilibrium, the firms chose to produce (Q_c, P_c) (see the figure), equating the private marginal cost with the competitive marginal revenue CMr. Notice that because we are in a competitive world, CMr is simply the price paid by the consumer $P_c = PMc$, which is independent of the quantity produced.

Up to (Q'_c, P'_c), increasing production impacts social welfare positively. In effect, for every $P > P'_c$ the marginal social benefit of increasing production P, is higher than the social cost SMc. Things change once production goes over Q'_c. Producing more at a lower price increases consumer surplus, but less

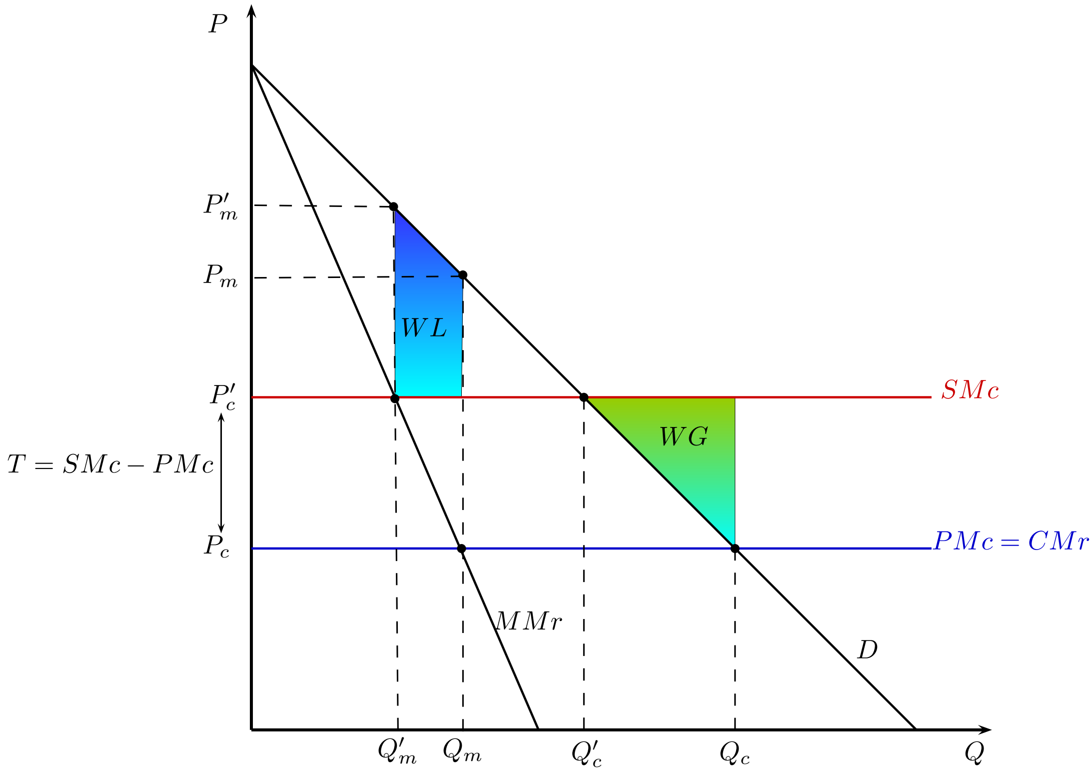

Figure 1: Pigovian taxes can be suboptimal with monopolies

than the decrease in producer's surplus and the increase in externalities as $SMc > D(Q)$ for $Q > Q'_c$. So producing the equilibrium quantity $Q_c > Q'_c$ induces a net welfare loss represented by the green triangle in the figure. In this case implementing a Pigovian tax $T = SMc - PMc$ is optimal, as it brings production back to Q'_c. The relative welfare gain with respect to the non-tax situation is represented by the green triangle WG. The Pigovian tax is optimal as any other tax level would lead to not producing some units which are socially beneficial, or to produce some units which are socially detrimental.

The story is much different in the case of a monopoly. Because the monopolist marginal revenue MMr is downward sloping, a monopolist will already produce less that the competitive level. It may then be the case as in the figure that the monopolist already produces some $Q_m < Q'_c$. As Q'_c was the threshold beyond which productions was socially detrimental, imposing the Pigovian tax $T = SMc - PMc$ would make things even worse. It would result in the monopolist producing $Q'_m < Q_m$ hence not producing some quantities which are socially beneficial. This adds another welfare loss to the loss induced by the market imperfection, represented by the polygon WL in the figure.

Buchanan's example has the merit to shed light on the important relation between market structures and externality regulation. In his article he concludes that "it is necessary to limit the Pigovian correctives on the tax side to situations of competition" (Buchanan, 1962). This claim deserves some attention as it may easily be misunderstood. The fact that any tax is detrimental for social welfare is an artifact of the special case that we studied. As can be seen from figure 2, when the externality is bigger, it may still be desirable to tax the monopolist. What is more, as suggested by Lee (1975), even a Pigovian tax may be better than no tax at all. In figure 2, this is illustrated by the fact that the welfare loss associated with no taxes WL_0 is bigger than the welfare loss WL_T associated with the Pigovian tax $T = SMc - PMc$ (these are welfare loss with respect to the first best production Q'_c). However, on can see that the Pigovian tax, is not optimal. Although a Pigovian tax is in this case better than nothing, it is worse than a slightly lower tax which could bring production back to the optimal level. So a proper way to read Buchanan's

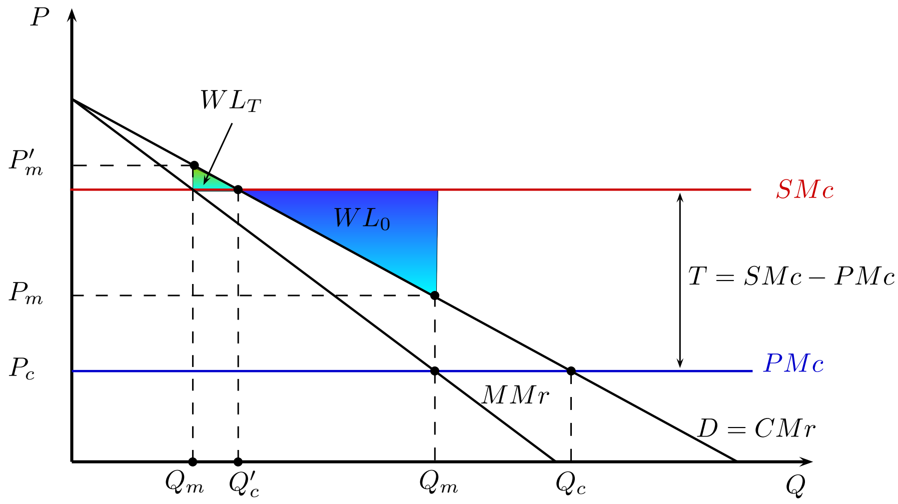

Figure 2: A Pigovian tax is still better than inaction

claim is "it is necessary to limit the Pigovian correctives on the tax side $T = SMc - PMc$ to situations of competition, as there are better tax levels $T' \neq T$ when competition is imperfect".

Determining precisely how and to which extend one can do better than Pigovian taxes is the question studied by Lee (1975) and Barnett (1980), and the topic of the next section.

2 How to do better than Pigovian taxes?

In this section, we cover the first attempts to devise optimal tax rules in non-competitive frameworks. The question is whether a different tax rule than the Pigovian one can do the job of efficiently regulating externalities, and if so what does this tax rule look like.

Lee (1975) was maybe the first to come to grips with this research agenda. Lee's framework differ in many respect from the setting of Buchanan (1962) :

- There are multiple firms producing the same good.
- Each firm faces its own demand function which only depend on the quantity it produces.
- Firms use multiple inputs.
- Among all the inputs, only one induces an externality and the government can tax this very input (government is not limited to taxing output).

For the sake of continuity, we will present the gist of Lee's argument while remaining as close as possible from the model of Buchanan (1962). The possibility of taxing input instead of outputs turns out to be only essential departure from Buchanan (1962). to the core of Lee's argument. This is therefore the only one we will adopt here. See Frame II for a glimpse at a the multi-firm model.

Suppose again that there is a monopolistic firm producing some good in a partial equilibrium setting. The firm only uses one input x to produce output q with the production function being

$$q = f(x). \tag{9}$$

The production function is twice continuously differentiable and concave, with decreasing returns past some threshold $\bar{x}$, i.e. $f'(x) < 0$ for all $x > \bar{x}$. The use of the input induces an externality. For simplicity, let us follow Lee (1975) and assume that the input x consists of effluent emission which induce negative externalities. As in Buchanan (1962), assume that $t > 0$ is the fixed marginal external cost of an additional unit of emission x.

The firm faces the inverse demand function

$$p = P(q), \tag{10}$$

which is non-increasing in q. Without any intervention, producing discharging effluents is free. When w the price of input x is larger than zero, it should therefore be interpreted as the tax on using input x.

The producer faces the classical maximization problem

$$\begin{aligned} max_x \quad & pq - wx \\ s.t. \quad & (9) \text{ and } (10) \end{aligned} \tag{11}$$

The first-order conditions (FOC) for an interior solution are

$$\frac{df(x)}{dx}\left[\frac{dp}{dq}q + p\right] - w = 0, \tag{12}$$

Let $\tilde{x}$ be the solution of (12) and let $\tilde{q} := f(\tilde{x})$. In our simplification of Lee's framework, the Marshallian surplus equation takes a slightly different form than (8) in Frame I. Because w is a tax, the use of inputs induces a third type of surplus in the form government revenues, or *government surplus*. Again, one may wonder whether government revenues should be included in the measure of social welfare. Ultimately, only consumers' welfare matters and whether the government raises positive taxes should be relevant only in as much as it influences consumers' welfare. So including government surplus in the Marshallian surplus is consistent only under the hypothesis that the government directly transfer all its revenues to the consumer in a lump-sum manner.

Let GS stand for government surplus. Then Marshallian surplus at equilibrium takes the form

$$\int_0^{\tilde{q}} P(q)\ dq - w\tilde{x} + GS - t\tilde{x} \tag{13}$$

Notice that government revenue is simply the taxes paid by producers to use the polluting input that is $GS = wx$. So the expression for Marshallian surplus simplifies to

$$\int_0^{\tilde{q}} P(q)\ dq - t\tilde{x} \tag{14}$$

If the equilibrium was social optimal, there should be no way to improve social welfare by altering the quantity of input x. That is the derivative of (14) with respect to x should be equal to zero when evaluated at $\tilde{x}$.

$$\begin{aligned} \frac{d}{dx}\left[\int_0^q P(q)\ dq - tx\right]\Bigg|_{x=\tilde{x}} &= 0 \\ P(\tilde{q})\frac{d\tilde{q}}{dx} - t &= 0 \\ P(\tilde{q})\frac{df(\tilde{x})}{dx} - t &= 0. \end{aligned} \tag{15}$$

The left-hand side of last equation gives the marginal social benefit of changing the level of input at an equilibrium. If it is zero for some w^*, then w^* can be said to *implement the social optimum in a decentralized equilibrium.* This means that given w^*, a decentralized market would achieve social optimality. Now because $\tilde{x}$ is a solution of (12) we have

$$\frac{df(x)}{dx} = \frac{w}{\left[\frac{dp}{dq}\tilde{q} + P(\tilde{q})\right]}.$$

Pluging this back in the social optimality condition (15) we obtain

$$P(\tilde{q})\frac{w}{\left[\frac{dp}{dq}\tilde{q} + P(\tilde{q})\right]} = t. \tag{16}$$

The last conditions tells us many things. First notice that (16), is really a generalization of the social optimality condition from the competitive case. When competition is perfect, $\frac{dp}{dq} = 0$. In the ideal case where $t = 0$, (16) then becomes

$$w = 0, \tag{17}$$

i.e. setting taxes to zero is optimal. If $t \neq 0$ but the market is still competitive, (16) becomes

$$w_{compet} = t, \tag{18}$$

that is Pigovian taxes are optimal. Notice that condition (18) holds whether t is larger or smaller than zero. In the case of a positive externality with $t < 0$, government surplus is simply negative and the lump-sum transfer of government surplus becomes a lump-sum tax meant to cover government's deficit.

In the general case $t \neq 0$ and $\frac{dp}{dq} < 0$, we find that the optimal tax on input is

$$\begin{aligned} w_{monop} &= t\frac{P'(\tilde{q})q + P(\tilde{q})}{P(\tilde{q})}, \\ &= t\left[1 + \frac{P'(\tilde{q})\tilde{q}}{P(\tilde{q})}\right], \end{aligned} \tag{19}$$

where $P'(\tilde{q}) := \frac{dP(\tilde{q})}{dq}$. The term $P'(\tilde{q})\tilde{q}$ which appears in (19) is a standard indicator of market power measuring the difference in terms of marginal revenues from a competitive firm . Because $P'(\tilde{q}) < 0$, we also have $\frac{P'(\tilde{q})\tilde{q}}{P(\tilde{q})} < 0$. Therefore, if we face a negative externality with $t > 0$, we will have $w_{monop} < w_{compet}$ (the reverse holds for a positive externality). In addition, the higher the monopoly power of the firm, the lower the optimal tax rate on its polluting input.

All this is in accordance with the graphical argument from Buchanan (1962). The cases in which no tax should be raised on the monopolist corresponds to situations in which

$$-1 = \frac{P'(\tilde{q})\tilde{q}}{P(\tilde{q})}.$$

If we went as far as having $-1 > \frac{P'(\tilde{q})\tilde{q}}{P(\tilde{q})}$ then it would be optimal to subsidize the monopolist despite the negative externality. Finally notice that the tricky bits in computing w_{monopo} empirically lies in the evaluation of t. Apart from the money value of the externality, computing w_{monopo} only requires estimating standard objects, namely the aggregate demand function and the equilibrium quantity.

The derivation of optimal tax rules under more sophisticated assumptions can be found in Lee (1975), Barnett (1980), as well as in Requate (2007) for a synthetic exposition (input vs. output taxation, with and without abatement technology, etc.). The exact formula for the optimal tax rate w^* will differ from one setting to another. Things become more complicated in particular when the externality is not linear (see Barnett (1980)). The general process of deriving optimal tax rates is however more or less invariant. It always consists in :

1. Finding the social optimal production level given the externality.
2. Finding the value of w which makes the privately optimal production plan align with the socially optimal one.

As in our example, the optimal tax rate usually depend on some index of firms' market power.

3 Alternative policy instruments : the case of tradable emission permits

In this section we investigate the cost-effectiveness of tradable permits in reducing emissions. We will mainly follow the analysis of Hahn (1984). The question of cost-effective emission reduction diminution is significantly different from the problem of maximizing Marshallian surplus via decentralized taxes. Cost-effective emission reduction is what is called a *second-best problem* because we do not worry about the desirability of the emission target. We assume that there is a certain amount of emission that must be reached, or at least not exceeded. This emission target is exogenous, taken as a constraint, and might not be optimal itself. The second-best question is then what is the most cost effective way to meet this emission constraint. In other words, we want to know what is the best we can do given the emission constraint.

Following Hahn (1984), we will in fact consider a *third-best problem* in which an additional constraint needs to be met. If permits are traded among producers, we will require that trade take place at a unique and common market price. As was pointed out by Lee (1975) this a demanding constraint. In general, if producers face different technologies and cost functions, it is better to require more emission reduction from the most effective firms. In the case of a emission market, this would mean charging each firms a different price for emission permits. Similarly when pollution is dealt with through taxation, different firms should face different taxes depending on their effectiveness at using the polluting input in the production the good. However, as Lee (1975) notices, this differentiating taxes based on costs might be politically unfeasible if not unconstitutional. It is then worth studying optimal policies under the constraint of a common price level. See Frame II for more on differentiated taxes.

Frame II : Cost-effective reduction of emission between multiple firms

The model. The setting is from Lee (1975) and is similar to the one we studied before, but with multiple firms. There are m firms labeled $\{1, \dots, m\}$, each producing the same good. Firm j's technology is defined by the production function

$$q_j = f_j(x_j). \tag{20}$$

The production function has the same properties as before. For concreteness again, let us study a negative externality with the monetary value of the externality produced by firm j being tx_j. Each firm is associated with an inverse demand function

$$p_j = P_j(q_j), \tag{21}$$

non increasing in q_j. Equation (21) allows firms to differ both in their production function and in their market power, each serving part of the market.[a] The prices of the input is fixed and denoted w. As before, producing the externality is initially free. When different from zero, w is again a tax on x_j.
Each producer's maximization problem is

$$\begin{array}{ll} max_{x_j} & p_j q_j - w x_j \\ & s.t. \quad (9) \text{ and } (10) \end{array} \tag{22}$$

Let $f'_j(x_j) := \frac{df_j(x_j)}{dx_j}$. The FOC for an interior solution is

$$f'_j(x_j)\left[\frac{dp_j}{dq_j} q_j + p_j\right] - w = 0, \tag{23}$$

In this multi-firm setting, the complete formulation of the Marshallian surplus takes the form

$$\sum_{j=1}^{m}\left[\int_0^{\tilde{q}} P^j(q_j)\, dq_j - w x_j + \underbrace{w x_j}_{Gov.Surpl.} - t x_j\right]. \tag{24}$$

Let $\tilde{x}_j$ be the level of firm j's input satisfying (24). If we were to look for the optimal tax level, we would need to worry about how much reduction in x_j is desirable given the benefit in terms of external damage. In a second-best framework things are easier. We are given a fixed emission target R. Whether the target R is optimal or suboptimal is not our business. We have to do our best in terms of Marshallian surplus given the constraint

$$\sum_{j=1}^{m} x_j = R \tag{25}$$

Simplifying (24) and using the constraint (25), the Marshallian surplus becomes

$$\underbrace{-t\sum_{j=1}^{m} x_j}_{=tR,\ cst.\ under\ (25)} + \sum_{j=1}^{m}\left[\int_0^{\tilde{q}} P_j(q)\, dq_j\right], \tag{26}$$

Cost-effective reduction of emission between multiple firms (continued)

Condition (26) makes it clear that under the emission constraint, we should only care about $\sum_{j=1}^{m}\left[\int_0^{\tilde{q}} P_j(q_j)\, dq_j\right]$ when maximizing the Marshallian surplus.

Differentiated taxes. Take any tax level w identical for every firm. We want to show that a uniform w is optimal only under very specific circumstances. To do so, consider the marginal social benefit of changing $\tilde{x}_j$, the level of input used by firm j at equilibrium. By deriving (26), this marginal social benefit is zero when

$$P_j(\tilde{q})\frac{d\tilde{q}_j}{dx_j} = P_j(\tilde{q}_j)f_j'(\tilde{x}_j).$$

By the FOC (23), the private cost of changing $\tilde{x}_j$ is given by

$$f_j'(\tilde{x}_j)\Big[\frac{dP(\tilde{q}_j)}{dq_j}\tilde{q}_j + P_j(\tilde{q}_j)\Big] = w. \tag{27}$$

Private and social cost are equal only in the competitive case in which firms have no market power and

$$\frac{dP(\tilde{q}_j)}{dq_j} = 0, \tag{28}$$

or when

$$f_j'(\tilde{x}_j)\Big[\frac{dP(\tilde{q}_j)}{dq_j}\tilde{q}_j + P_j(\tilde{q}_j)\Big] = f_k'(\tilde{x}_k)\Big[\frac{dP(\tilde{q}_k)}{dq_k}\tilde{q}_k + P_k(\tilde{q}_k)\Big], \qquad \text{for all } j,k \in \{1,\dots,m\}. \tag{29}$$

In this second case, it is equally desirable to reduce the production of any of the firm. By chance, all firm happen to have the same private price of reducing emission at equilibrium. Then assuming that the emission constraint is binding, the equilibrium allocation of production among firm is a second best optimum because no redistribution of production reduces costs.
Only the second case features imperfect competition and it should clear that it is highly unlikely. In general, one cannot reach a second-best equilibrium by imposing the same tax level on each firm. Second-best optimality requires differentiated taxes, with higher taxes on the most cost-effective firms. These may be shown to be equal to

$$w_{multi} \coloneqq t + f_j'(\tilde{x}_j)\frac{dP(\tilde{q}_j)}{dq_j}\tilde{q}_j, \qquad \textit{see Lee (1975)},$$

that is the optimal tax level differ from the Pigovian tax t by a firm-specific factor depending on the market power and the marginal productivity of each firm.

[a] In these notes, we do not consider explicit models of competition. See Requate (2007) for an extensive discussion of externality regulation in different competition model including Cournot and Bertrand competitions.

In a nutshell, Hahn (1984) stresses that the competitiveness of the market for inputs is just as important as the competitiveness of the market for outputs in order to achieve cost-effective emission reduction. As such, the fact that imperfect competition in the input market induces inefficiencies is no surprise and is not specific to the case of firms producing externalities. What is more interesting is the fact that the distribution of endowments can be used to alleviate efficiencies in markets with imperfect competitions. Intuitively, one can reduce inefficiencies by allocating endowments in order to anticipate the demand of firms with market power. At the limit, if firms with market power are precisely assigned their equilibrium demand, they will not participate in the market. As a consequence, they will not use their market power and the competitiveness of the market will be restored.

This last point allows a quick discussion of "Coase theorem". In its weakest interpretation, Coase theorem is sometime formulated as

Weak Coase Theorem 1 : If there are not transaction costs, then any initial assignment of the emission rights leads to a Pareto optimal allocation if the agents have the opportunity to trade their property rights. (See lecture notes)

If permits (i.e. property rights) are traded via a market, the Weak Coase Theorem 1 is in essence an application of the First Welfare Theorem. Hahn (1984) draws our attention on the fact that, as any First Welfare Theorems, Coase Theorems requires that agents behave as if they were price takers. The initial formulation "Weak Coase Theorem 1" is therefore not complete. A more complete formulation is:

Weak Coase Theorem 2 : If there are not transaction costs, then any initial assignment of the emission rights leads to a Pareto optimal allocation if the agents have the opportunity to trade their property rights on a *competitive* market.

As noticed in the lecture notes, this might look like a cheap shot. Relaxing the constraint that permits be traded via a market, Weak Coase Theorem 2 boils down to a tautologies of the form

Weak Coase Theorem 3 : if permits are traded via a mechanism which is Pareto efficient *for any initial assignment of the endowment*, then the outcome of the mechanism is Pareto efficient *for any initial assignment of the endowment*.

The content of Coasian research agenda's then reduces to a search for Pareto efficient trade mechanism whose efficiency is independent of initial endowment. Then, the only interest of studying the special case of permit trading is the concreteness of the notion of a social endowment. In most application, the idea that an social endowment must be distributed by a planner between the agents before they start trading is unrealistic. It is quite the contrary when it comes to permit markets.

Hahn (1984) model can be interpreted as follows. Suppose there are m firms producing the same good q. The market for q is competitive so that selling price p is taken as given. Firm j's technologies is defined by the cost function $C_j(q_j, e_j)$ where e_j is the quantity of emissions firm j is allowed to produce. Realistically, ${}^{dC_j}/_{dq_j} > 0$ and ${}^{d^2C_j}/_{d^2q_j} > 0$. Emissions levels e_j can be viewed as emission permits. If a firm buys an additional emission permit, its cost decreases because it does not have to pay for emission abatement on this emission unit. Therefore, $-{}^{dC_j}/_{de_j} > 0$ can be viewed as the abatement cost of a unit of emission. It is also assumed that $-{}^{d^2C_j}/_{d^2e_j} > 0$.

To buy an emission permit, the firm must go on the permit's market and pay the equilibrium price for permit P. Firms are also endowed by the government with a initial quantity of permits e_j^0 which they can sell on the market at the same equilibrium price. Now consider firm j's profit maximization problem

$$\max_{q_j, e_j} \qquad pq_j - C_j(q_j, e_j) - P(e_j - e_j^0).$$

The FOC for q_j is

$$p = \frac{\partial C(q_j, e_j)}{\partial q_j},$$

which for a fixed p implicitly defines q_j as a function $q_j(e_j)$ (remember that we assume that the market for outputs is competitive). So the problem can be rewritten as

$$\max_{e_j} \qquad -C(q_j(e_j), e_j) - P(e_j - e_j^0),$$

which given $\tilde{C}_j(e_j) \coloneqq C_j(q_j(e_j), e_j)$ is equivalent to

$$\min_{e_j} \qquad \tilde{C}_j(e_j) + P(e_j - e_j^0). \tag{30}$$

Let us now turn to the description of the emission market. Firms 2 to m are price takers on the emission market. Given the price of emission rights P, they solve (30), which yields the FOC

$$\tilde{C}_j'(e_j) = P. \tag{31}$$

Notice that $\tilde{C}$ inherits the second derivative properties of C. Therefore, the last equation implicitly defines firm j's downward sloping demand for input e_j which we denote $e_j(P)$. For a price taker firm, the demand for permits is independent of the endowment, as was to be expected from Weak Coase Theorem 2.

Firm 1 on the other hand has complete market power in the sense that it can set the price at which permits are exchange. Firm 1's only constraint is that, given firms' demand (including it's own), it must set the price to be market clearing, the supply of permits being simply the sum of the endowment $L = \sum_{j=1}^{m} e_j^m$. Formally, firm 1's problem is

$$\min_{P} \quad \tilde{C}_1(e_1) + P(e_1 - e_1^0)$$

$$s.t. \quad e_1 = L - \sum_{i=2}^{m} e_i(P).$$

Directly substituting the constraint in the maximand, one obtains the FOC

$$-\tilde{C}_1'(e_1)\sum_{i=2}^{m} e_i'(P) + (e_1 - e_1^0) + P\sum_{i=2}^{m} e_i'(P) = 0 \tag{32}$$

From (32) one can readily see that contrary to firms 2 to m, firm 1's optimal choice does depend on its endowment e_1^0 in general. More specifically, if the monopolist's endowment precisely equals its demand $e_1 = e_1^0$, we have

$$-\tilde{C}_1'(e_1)\sum_{i=2}^{m} e_i'(P) + P\sum_{i=2}^{m} e_i'(P) = 0$$

$$\left[\sum_{i=2}^{m} e_i'(P)\right]\left[-\tilde{C}_1'(e_1) + P\right] = 0. \tag{33}$$

Given the assumptions on C, the last equation only holds when

$$\tilde{C}_1'(e_1) = P, \tag{34}$$

the same condition as for the firms with no market power (one can see from (32) that (34) does not hold for $e_1 \neq e_1^0$). Only in this very special case is the marginal abatement cost equal among all firms

$$\tilde{C}_j'(e_j) = \tilde{C}_k'(e_k), \qquad \text{for all } j,k \in \{1,\dots,m\},$$

which is the traditional first order social optimality condition for cost-minimization problems (remember the discussion of Frame II). As alluded to above, this result leads itself to an easy interpretation. If $e_1 = e_1^0$, the firm with market power does not actually participate in the market. It simply consumes its endowment of emission permits and has a zero net trade. Therefore, it does not effectively use its market power to steer market prices. It has no reason to do so as it will not buy anything on the market. Therefore setting $e_1 = e_1^0$ allows to recover optimality. More generally, one can show that the further (below or above) the equilibrium e_1 is from the endowment e_1^0 the larger the cost inefficiency of emission abatements (see Hahn (1984)).

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